ADRRI JOURNAL OF PHYSICAL AND NATURAL SCIENCESE
ISSN: 2026-5360
VOL. 4, No. 2 (4), July, 2021-September,2021
PUBLISHED BY AFRICA DEVELOPMENT AND RESOURCES RESEARCH INSTITUTE

ADRRI JOURNAL OF PHYSICAL AND NATURAL SCIENCES

*ADRRI JOURNALS (***www.adrri.org***)*

E-ISSN: 2026-5360 VOL. 4, No. 2 (4), July, 2021-September, 2021

An integrated supply chain inventory model for imperfect quality items when procurement cost is linked with trade credit policies

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Available Online: 30th September, 2021

URL: https://www.journals.adrri.org/

Abstract

This study develops an economic production model for manufacturer and retailer with credit-linked procurement cost. Here buy now and pay later policy is offered by manufacturer to retailer. In this model retailer's procurement cost is linearly dependent on the credit period offered by the manufacturer. Quality of a product plays a very important role to attract buyers for a product. To increase the profit industries are always working on producing a good quality product. Manufacturer's process cost is also dependent on the quantity demanded by the retailer. The lot received by retailer contains imperfect quality items and items that could get deteriorated with time. It is assumed that rate of screening is more than demand so as to fulfill the demand by good quality products only. Shortages are not allowed. The model is explained with the help of numerical example and sensitivity analysis of some parameters.

Keywords: procurement cost, imperfect quality, trade credit, supply chain management, inventory

[**Cite Article as:** Gilotra, M., Dhaka, V. and Supriya, (**2021**). **An integrated supply chain inventory model for imperfect quality items when procurement cost is linked with trade credit policies.** ADRRI Journal of Physical and Natural Sciences, Ghana: Vol. 4, No. 2 (4). Pp. 30-40, **E-ISSN**: 2026-5360, 30th September, 2021.]

Received: (May 15, 2021)

Accepted: (September 30, 2021)

INTRODUCTION

Now days, supply chain management is a popular practice in manufacturing systems and inclusion of trade credit policies plays a vital role in this supply chain. Under the context of supply chain inventory models, it is assumed that the price for the ordered quantity is paid when the order is placed. But, in today's world this is not always true in business world. Retailers in general follow the practice of buy now and pay later to settle the accounts. No interest is charged from retailers on the amount owned during the credit period. However, if the account is not settled by the end the delay period interest will be charged. This proves to be more beneficial for the retailers as they don't have to pay immediately after receiving the shipment. They can pay till the end of the delay period.

Goyal (1985)proposed an economic order quantity model with some trade credit policies, i.e. buy now pay later. Aggarwal and Jaggi (1995)suggested an EOQ model for determining the order quantity with deteriorating items following some trade credit policies. Conventional inventory models usually theorize that the amount is paid by the retailer to the supplier amid deal. But in reality, a supplier generally offers the retailer some time extension to pay thereby resulting in increment of ordering quantity. During this time period the retailer is under pressure to pay and can also earn the interest from deposited sales revenue. Das et al., (2013) consider a supplier-retailer production inventory system of deteriorating item with variable trade credit policy in a finite time horizon for known demand.

Generally, in economic production quantity models, it is assumed that the quality of product and its manufacturing process are perfect. This assumption is far from the reality, product quality may not be always perfect. The production process may be subject to deterioration due to the occurrence of one or more assignable causes. The occurrence of the assignable causes may shift the process from a state-in-control to a state-out of control and produce some defective items. Imperfect items in the raw material and production stages directly impact the coordination of the product flow. In response to this concern, production and inventory lot sizing models, which incorporate imperfect items into their formulation have become an important and growing area for researchers.

Porteus (1986) combined the presence of imperfect quality items into the basic economic order quantity model. He presumed there is a chance $q$that the production process may go out of control while manufacturing one unit of the product. Salameh & Jaber, (2000)prolonged the performance of EOQ model with the presence of defective items under random yields which conflicts the result of Rosenblatt & Lee, (1986)that the economic lot size quantity is inversely proportional to the presence of defective items. Cardenas-Barron (2000) modified the "optimal order quantity expression" and Goyal & Cárdenas-Barrón (2002) recommended a real-world approach on economic order quantity for defective items. Papachristos & Konstantaras (2006)observed the matter of non-shortages in model with relative defective quality items, when the quantity of these defective items is a random variable. They identified that the sufficient conditions given in the Salameh & Jaber, (2000)paper to prevent shortages may not really be so useful. Maddah and Jaber, (2008) rectified the limitation in their EOQ model with inaccurate stock, categorized by a random fraction of defected quality items and a screening process. Jaggi et al., (2011) suggested an inventory model for deteriorating items with the presence of defective items following some trade credit policies.

In this paper an inventory model for deteriorating and imperfect quality items when procurement cost is linked with trade credit policies is developed. Here Retailer's procurement cost is linearly proportional to the credit period. The processing cost of the supplier contains two parts – (i) A definite amountQ_0 of items bought by the retailer is for a particular price. and (ii) the other price increases as the quantity exceeds the certainQ_0 of items. The supplier gives a delay in time period M for the payment to the retailer and later charges the interest for the non-selling amount after the delay period. The Supplier does not charge any interest nor the payment is taken for the purchased items before the end of replenishment period T. Interest charges is only paid exactly at the time T.The interest reduction or discount up-to Mattracts the retailer.The given fig 1 (Das et al., 2013) shows the entire process for the modelling. The main objective is to find the effect of defective and deteriorating items on the total cost of the supply chain.

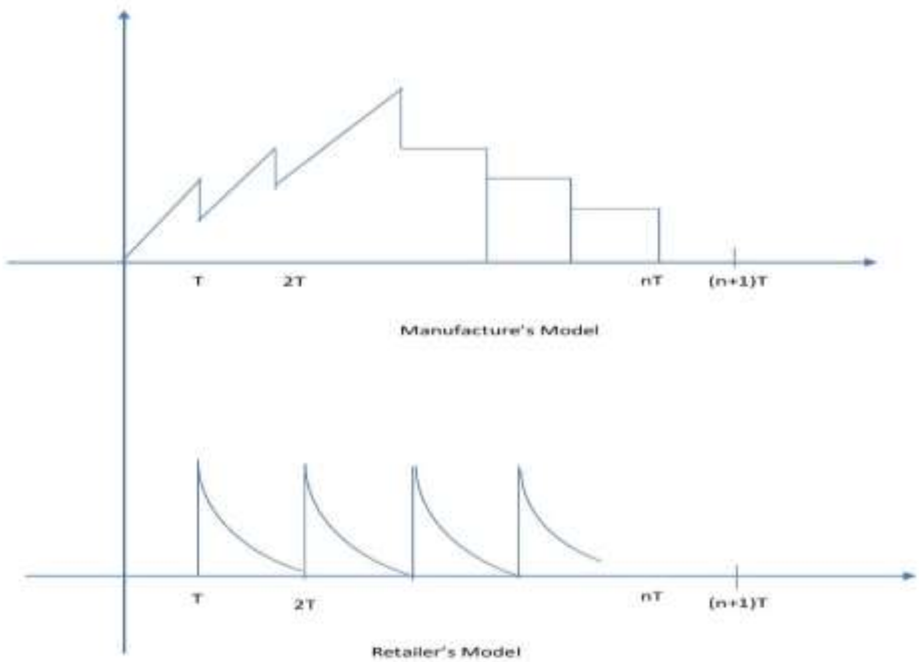

Fig: Inventory Model

Assumptions and Notations

Notations ((Das et al., 2013) and (Jaggi et al., 2011))

In this integrated inventory model, the following notations are used:

A: retailer's ordering cost per order.

S: supplier's setup cost per production run.

V: variable process cost to the supplier of dealing with the retailer.

c: retailer's procurement cost per unit item.

u: supplier's production cost per unit item.

h_r: retailer's inventory holding cost rate, excluding interest charge.

h_s: supplier's inventory holding cost rate.

p_r: retailer's selling price per unit item.

D: demand rate of the customer to retailer.

P: production rate of the supplier.

θ: deterioration rate of the item for the retailer only.

T: replenishment time interval of the retailer in year unit (decision variable).

M: credit period offered by the supplier (decision variable) to the retailer.

I_d: interest rate of revenue deposited by retailer.

I_c: interest rate to be paid to the supplier for the remaining stock from M to T.

I_s: interest rate for calculating supplier's opportunity interest loss due to delay payment.

n: No. of replenishment of retailer.

Q: initial quantity which is taken by the retailer for a cycle from the supplier.

TR_1, TR_2: average total cost of the retailer for deteriorating and non-deteriorating item respectively.

$TS_1 TS_2$: average total cost for the supplier for deteriorating and non-deteriorating item respectively.

TC_1,TC_2: average total cost of retailer and supplier together for model with and without deterioration respectively.

Assumptions

To develop the proposed integrated model for supplier and retailer the following assumptions are made:

- The model deals with a single supplier and single retailer for a single product.
- The replenishment is instantaneous for retailer.
- The supplier produces the item and then fulfils the retailer's demand simultaneously, so at the beginning of production of item, there is very small possibility of deterioration in general. Moreover, supplier is a big merchant who can have capacity to prevent deterioration. So, in this model, deterioration is considered for retailer only at the rate of h, is assumed to be constant over the business period.

- The screening and demand proceeds simultaneously, but the screening rate (λ) is greater than demand rate $(D), \lambda > D$.
- The defective items are independent of deterioration.
- The defective items exist in lot size (Q) and the percentage defective $(\alpha)$is arandom variablehaving uniform p.d.f. as $f(\alpha)$ with expected value

$$E(\alpha) = \int_{a}^{b} \alpha f(\alpha) d\alpha\,, \qquad 0 < a < b < 1.$$

- The screening rate (λ) is sufficiently large such that screening time (t_1) is always less than the permissible delay period (M), i.e. $t_1 \leq M$and $t_1 = (Q/\lambda) \leq T$. In general, this assumption should be acceptable since the automatic screening machine usually takes only little time to inspect all items produced or purchased.
- Demand rate (D) and production rate (P) are assumed to be constant but P is greater than D.
- Shortages are not allowed.
- In this production-inventory system, the whole business period is assumed to be one year, i.e., time horizon is finite.
- It is assumed that the credit period (M) offered by supplier must be within each replenishment period (T) (i.e., $M \leq T$).
- Supplier charges an interest at the rate of I_c on the remaining amount of stock after the credit period M.
- Retailer's procurement cost (c) linearly depends on the credit period (M) by the relation $c = c_0 + c_1 M$, where c_0 is the procurement cost in the absence of credit period and $c_1 > 0$.
- Supplier's process cost depends linearly on the quantity purchased by the retailer by the relation $V = V_0 + V_1(Q - Q_0)$, where V_0 is the fixed process cost for the amount purchased by retailer which is less than and equal to Q_0.
- Since it is an integrated model so the system i.e., retailer and supplier desire to settle the credit period M and replenishment period T in such a way that the system cost is minimum. So, in this paper M and T are considered as decision variables.

Formulation for Supply chain Process

The manufacturing by supplier starts at $t = 0$ at P rate. Initially the retailer receives its first shipment of Q number of items after the replenishment time T. The demand of customers is fulfilled by retailer from the stock in the time period T. After time T supplier sends the same quantity of items Q to the retailer. This procedure goes on and on till nT, n being the number of replenishments and at the time $(n + 1)T$, the stock of the retailer vanishes.

Modelling

Since, the inventory scenario of retailer for each cycle is same, hence the inventory level and all costs will be same for all the cycle. Now the length of each cycle is T. So, below are the different cost associated with the model.

For retailer the differential equation of $I(t)$ for each replenishment cycle will be:

$$\frac{dI(t)}{dt} + \theta I(t) = -D, \quad 0 \le t \le T$$

With boundary condition

$$I(0) = Q \; and \; I(T) = 0$$

Solving the above differential equation

$$I(t) = Qe^{-\theta t} + \frac{D}{\theta}\left(e^{-\theta t} - 1\right), \; 0 \le t \le t_1$$

Inventory level at time t_1, including the defective items is

$$I(t_1) = Qe^{-\theta t_1} + \frac{D}{\theta}\left(e^{-\theta t_1} - 1\right)$$

After the screening process, the number of defective items at time, t_1 is αQ.

Hence, the effective inventory level during $t_1 \le t \le T$ is given by

$$I(t) = Qe^{-\theta t} + \frac{D}{\theta}\left(e^{-\theta t} - 1\right) - \alpha Q, \qquad t_1 \le t \le T$$

Therefore at $t = T$ and $I(t) = 0$, order quantity is follows as

$$Q = \frac{D\left(e^{\theta T} - 1\right)}{\theta(1 - \alpha e^{\theta T})}$$

Ordering cost: $OR_r = \frac{A}{T}$

Screening cost:$Sc_r = \frac{\beta Q}{T}$

Holding cost will be

$$HC_r = \frac{h}{T}\left\{\int_0^{t_1}\left(Qe^{-\theta t} + \frac{R}{\theta}\left(e^{-\theta t} - 1\right)\right) + \int_{t_1}^{T}\left(Qe^{-\theta t} + \frac{R}{\theta}\left(e^{-\theta t} - 1\right) - \alpha Q\right)\right\}$$

Interest earned by the retailer is divided into two parts

Part 1: Interest earned till the time period M

$$IE_1 = \frac{p.Ie}{T}\int_0^M R.tdt$$

Part 2: Interest earned on the defective items during the time period $M - t_1$

$$IE_2 = \frac{ps.Ie.\alpha.Q.(M - t_1)}{T}$$

Interest Paid by the retailer is:

$$IP = \frac{c.Ip}{T}\left(\int_M^T\left(Q.e^{-\theta t} + \frac{R}{\theta}\left(e^{-\theta t} - 1\right) - \alpha Q\right)dt\right)$$

Total cost for retailer's model

$$TC_r = OR_r + Sc_r + HC_r + IP - IE_1 - IE_2$$

$$= \frac{A}{T} + \frac{\beta Q}{T}$$

$$+ \frac{h}{T}\left(-\frac{Qe^{-\theta t_1}\theta + Rt_1\theta + e^{-\theta t_1}D - Q\theta - D}{\theta^2}\right.$$

$$\left.- \frac{\alpha QT\theta^2 - \alpha Qt_1\theta^2 + Qe^{-\theta T}\theta - Qe^{-\theta t_1}\theta + DT\theta - Dt_1\theta + De^{-\theta T} - De^{-\theta t_1}}{\theta^2}\right)$$

$$+ \frac{1}{T\theta^2}\left(cIp(\alpha QM\theta^2 - \alpha QT\theta^2 + MD\theta + Qe^{-\theta M}\theta - Qe^{-\theta T}\theta - DT\theta + e^{-\theta M}D - e^{-\theta T}D)\right)$$

$$- \frac{1}{2}\frac{pIeDM^2}{T} - \frac{ps.Ie.\alpha.Q.(M - t_1)}{T}$$

Supplier's Model

Das et al., (2013) proposed that "supplier's inventory starts from initial time $t = 0$ and stock of the supplier will be vanished at time nT, when retailer receives last replenishment of the item. The supplier continues the production up-to time n_1T [where $n_1(< n)$ is a positive integer]. Therefore, the inventory level of supplier increases up-to this time and after every cycle time T, the inventory level decreases instantaneously of amount Q due to demand (D) of the retailer."

Supplier's average inventory:

$$= \frac{Q}{2}\left[\left(n + 1 - \frac{nQ}{PT}\right)\right]$$

Cost for this average inventory will be

$$SI = \frac{h_rQ}{2}\left[\left(n + 1 - \frac{nQ}{PT}\right)\right]$$

Supplier's production cost

$$SP = \frac{uQ}{T}$$

Supplier's Opportunity Interest loss

$$SOIL = \frac{I_s cMQ}{T}$$

Total cost for supplier's model

$$TC_s = Set\ up\ cost + Process\ cost + SI + SP + SOIL$$

$$= \frac{S + nV}{nT} + \frac{h_rQ}{2}\left[\left(n + 1 - \frac{nQ}{PT}\right)\right] + \frac{uQ}{T} + \frac{I_s cMQ}{T}$$

Total cost for the complete model

$$TCC = TC_s + TC_r$$

$$= \frac{S+nV}{nT} + \frac{h_r Q}{2}\left[\left(n+1-\frac{nQ}{PT}\right)\right] + \frac{uQ}{T} + \frac{I_s cMQ}{T} + \frac{A}{T} + \frac{\beta Q}{T}$$

$$+ \frac{h}{T}\left(-\frac{Qe^{-\theta t_1}\theta + Rt_1\theta + e^{-\theta t_1}D - Q\theta - D}{\theta^2}\right.$$

$$\left.- \frac{\alpha QT\theta^2 - \alpha Qt_1\theta^2 + Qe^{-\theta T}\theta - Qe^{-\theta t_1}\theta + DT\theta - Dt_1\theta + De^{-\theta T} - De^{-\theta t_1}}{\theta^2}\right)$$

$$+ \frac{1}{T\theta^2}\left(cIp(\alpha QM\theta^2 - \alpha QT\theta^2 + MD\theta + Qe^{-\theta M}\theta - Qe^{-\theta T}\theta - DT\theta + e^{-\theta M}D - e^{-\theta T}D)\right)$$

$$- \frac{1}{2}\frac{pIeDM^2}{T} - \frac{ps.Ie.\alpha.Q.(M-t_1)}{T}$$

Numerical Example

The parametric data used for the mathematical experiment of the proposed model $D = 1000\, unit/year$, $P = 3{,}200\ unit/year$, $c = 100{,}000\, \$/year$, $A_v = 400\ \$/cycle$, $A_b = 25\ \$/cycle$,, $H_b = 5\ \$/unit$, $H_v = 4\ \$/unit$, $v = 2\ \$/unit$, $\lambda = 175{,}200\, unit/year$, $\beta = 0.5\, \$/unit$, $c_a = 200\, \$/unit$, $c_r = 50\, \$/unit$ and α is supposed to be distributed uniformly with its p.d.f is, $f(\alpha) = \begin{cases} \frac{1}{0.04-0}, & 0 \le \alpha \le 0.04 \\ 0, & otherwise \end{cases}$. Substituting these values in Eq we get the total cost of supply chain: $106555.312

CONCLUSION

In this paper, an inventory model is formulated considering the effect of deteriorating items and imperfect quality items on the supply chain when the trade credit policy used is different. The Supplier does not charge any interest nor the payment is taken for the purchased items before the end of replenishment period T. Interest charges is only paid exactly at the time T. The interest reduction or discount up-to Mattracts the retailer. There is an increment in the final cost of the supplier as well as the retailer because of the presence of imperfect quality items. Presence of imperfect quality items and deteriorating items could lead to more demand from the retailer. For future research rework shortages, Carbon emission as well as salvage value can be added for more realistic modelling.

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