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Application of Bühlmanns-Straub Credibility Theory in Determining the Effect of Frequency-Severity on Credibility Premium Estimation

Anani Lotsi[1], Felix Okoe Mettle[2] and Paul Kwame Adjorlolo[3]

[1,2,3] Department of Statistics and Actuarial Science, University of Ghana, legon

[1]Email: alotsi@ug.edu.gh, [2]Email: fomettle@ug.edu.gh,

[3]Graduate Student, University of Ghana

Email: padjorlolo@gmail.com

[1]**Correspondance**: alotsi@ug.edu.gh; (050) 569-4550

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Abstract

The study sought to demonstrate how credibility claim costs without the consideration of claim frequency and claim severities underlined by different risk profiles underestimate claim costs or premiums charged policyholders by non-life insurance companies. We used secondary data of non-life marine insurers in Ghana, claim histories that range from the period of 2013 to 2018. The claim histories included claim sizes, claim counts and policy counts. Bühlmans-Straub Credibility theory model was used in estimating credibility weights, credibility claim costs, credibility claim frequencies and credibility claim severities and subsequently find the credibility frequency-severity claim cost as the product of credibility claim frequency and severity for the individual and respective risk classes. We compared the estimates of the claim costs or premiums and have observed that the credibility claim costs underestimates claim costs or the average claim costs compared to the credibility frequency-severity claim costs for most of the risk classes. This is an indication of how a lack of consideration for variability or unstableness of claim frequencies and severities with different risk profiles undermine claim costs estimated through credibility rate makings in insurance. The study recommends that credibility ratemaking by insurance companies based on inadequate claim history and or with enough class risk variation should include credibility risk frequency and severity for the determination of credibility risk premiums.

Keywords: Bühlmans-Straub's Credibility theory, credibility claim cost, credibility frequency-severity, claim sizes, policy counts, claim counts and rate makings

INTRODUCTION

The non-life insurance companies like any other type of insurance business do not only act as risk managers but also serve as intermediaries that mobilize funds for banks which are also lend to business firms who also invest in the economy. The economy of Ghana like any other country will be affected if insurance companies do not charge risks the right premium in a competitive insurance business because insurance companies would not have enough funds to pay for losses or enough to invest in the economy (Mazviona, Dube, & Sakahuhwa, 2017). This can see many insurance companies out of business and the collapse of the insurance business can be disastrous to any economy, particularly in developing countries. As a consequence of these problems faced by insurance companies especially when there is little claim history available it is necessary to determine the best procedures of estimating fair premiums to be charged policyholders by these insurance companies.

Bühlmanns-Straub Credibility model is often employed in the field of Actuarial science to estimate fair premiums for future policyholders within different but similar classes of risks based on claim histories of such classes of risks (Centeno, 1989). Credibility techniques make use of collective claim histories and subsets of these collective claim histories. These techniques are useful in instances when there are limited claim histories or large claim histories but the characteristics of the collective claim histories are more heterogeneous compared to the claim histories of their subsets (Asamoah, 2016).

Pure premium or average claim cost is directly proportional to the product of claim frequency and severity. In this instance, underwriters assume that variations within annual claim costs, annual claim frequencies and annual claim severities over the accounting periods of the claim histories and across class risks are stable. Average claim costs (pure premiums) are charged policyholders and potential policyholders for the next immediate accounting date as minimum cost of their risks given that the variation in the claim histories over accounting periods remains constant. However, in insurance, where different risks are faced with different risk profiles, the issue of constant variation is rare and may undermine premiums to be charged for subsequent periods. In attempt to tackle this problem few researchers adopted and applied credibility models to annual claim costs whilst few others also tried to determine credibility models that best estimate claim costs as credibility premiums or claim costs. However, the neglect of variability in claim frequencies and claim severities in determining credibility estimates undermines credibility premiums (Hewitvitt, 1971). According to Kagen (2018) and Insurance Institute of Michigan (2019), unstable claim frequencies and severities significantly undermine pure risk premiums and indirectly affects claim reserves, liquidity of insurance companies and also denied insurers of premium for investment and also sometimes lead to adverse or anti-selection.

Despite the significant effort by many researchers to determine credibility estimates that do not undermine the pure risk premium or average claim cost (given limited claim histories), actuaries or researchers considers credibility claim cost without incorporating credibility frequency and severity which according to Hewitvitt, (1971) undermines pure risk premiums. Hence, the researchers through this study seek to estimate and compare credibility frequency-severity claim costs as a better and an alternative estimate to credibility claim costs.

The primary target of this study is to find the credibility frequency-severity premiums or claim costs for policyholders for each of the non-life marine insurance companies (risk classes) using claim frequencies and severities experienced by applying Bühlmans-Straub Credibility Theory model and also compare the result with credibility claim cost or premiums using annual claim costs estimated based on the same method and condition. The specific objectives of the study are to;

1. Estimate the average claim costs of the individual non-life marine insurance companies
2. Estimate the factors of credibility for each non-life insurance companies
3. Estimate credibility premiums based on claim costs and credibility frequency-severity premium for policyholders of the risk classes or individual non-life marine

insurance companies for the period 2019 using claim histories between the period of 2013 to 2018.

4. Compare the two credibility estimates; the credibility frequency-severity risk claim costs and the credibility risk claim cost.

The three questions that seem crucial in achieving the objectives of the study include;

1. How much variation is there in the claim costs or frequencies or severities of the companies?
2. How homogeneous are the risk classes in terms of claim frequency and claim severity?
3. How significant are the credibility factors of the Bühlmans-Straub model in doing experience rating?

In this paper, we employed the annual claim costs, annual claim frequencies and severities obtained or calculated from the annual aggregate claim amounts, volume of policyholders per year and number of claims made by the policyholders per year to calculate credibility estimates and respective weights. Specifically, we estimated credibility claim frequency and credibility claim severity and credibility frequency-severity claim cost obtained as a product of the two estimates (i.e. credibility claim frequency and credibility claim severity) using marine insurance claim histories of non-life marine insurance companies in Ghana. We then compared the credibility claim cost to the credibility frequency-severity claim costs. This is to demonstrate that estimating credibility of claim frequency and severity separately and independently before their product gives a better estimate compared to the credibility estimates by the claim costs.

Marine Insurance is a non-life insurance intended to cover losses or damages to ships, cargo, terminals and any other type of ocean transport for which property is transferred, obtained or kept between the point of origin and the final destination and also protects importers and exporters against loss through theft or damage to goods or property conveyed by sea including other waterways and rivers (Zogbenu, 2018). The claim histories of some non-life marine insurance companies in Ghana used in this study are only used as a case study to demonstrate a better way of determining fair premium with regard to credibility theory but not to analyse the financial position of any of these companies.

In an investigation, Laryea (2016) found that few researchers proposed different methodologies for accurately estimating insurance companies ' rates needed to charge policyholders in order to be able to cover outstanding claims in full when hazards occur. The actuarial accuracy measures of these distinct methodologies are based on the data available for claims and what assumptions are allowable. Drewry (1998), Haiss (2006) and Marrewijk (2002) all, in their study have made significant attempt to enumerate some problems and the causes undermining the performance and the profitability of the non-life insurance industry. In order for insurance companies to have enough fund to invest and also cover claims they faced, King (2008) suggested that non-life insurance industries must charge competitive and reasonable premiums as a solution to the problems of non-life insurance.

Zhang (2004) suggested a Bayesian non-linear hierarchical model that tends to tackle some of the major problems faced by insurance companies in anticipating the outstanding claim sizes for which they will finally be required to have in their books. This approach requires taking into account previous understanding and expert or technical opinion included in the assessment by judgmentally selected priori distributions. Viaene et al., (2007) similarly, in their investigation, found out that the assessment of the likelihood and claim is critical since a loss protection company can use these assessments to allocate claim reserve boundaries and discounts based on the risk features of an individual client. Consequently, Meulbroek (2000) argued that loss protection Companies must treat corporate governance as development of associated variables and opportunities. But Weisberg(1983), stated that insurance organizations are endeavouring to assess sensitive expenses of insurance arrangements depending on the announced losses for specific types of hazards. The credibility estimates for the claim costs based on claim sizes and policy counts are often the easiest way of setting premiums for class risks with limited claim data. However, the credibility claim cost without claim frequency and severity undermines the estimates because the claim sizes for the collective risk classes are underlined by a risk profile which is not the same for claim frequencies and claim severities just as the risk profiles are not the same for claim frequencies and severities. Therefore, the claim frequencies and severities, the two determinants of claim sizes or size of insurance losses must be estimated independently according to their respective risk profiles based on which the premiums should be estimated.

Here is the outline of the paper. In Section 2, we review related literature on the topic. Section 3 considers the methodology; data and source, Data structure and

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classification are defined. Also, Assumptions under Bühlmann-Straub's Credibility model, proof and estimation of model parameters as well as model assumptions for claim frequencies are all stated. Section 4 considers data analysis and numerical computation of the credibility parameter estimates, the credibility factors, credibility claim costs, credibility claim frequency, credibility claim severity and credibility frequency-severity claim costs or premiums. Finally, discussion and conclusions presented in Section 5 and section 6 respectively.

METHODOLOGY

Data

The study is an empirical application of claim histories acquired from the National Insurance Commission (NIC), the body mandated by way of law to ensure high quality administration, supervision, formulation of regulations and the management of insurance commercial enterprises in Ghana. The claim data constitute the claim sizes, policy counts and claim counts recorded on groups of policyholders per year class of risks or marine insurance companies reported to the National Insurance Commission by 15 of these non-life marine insurance companies in Ghana. The data covered the period, 2013 to 2018 for each risk company. From the claim histories we obtained the annual claim cost, annual claim frequency and severity for policyholders of each company.

Data Structure and Classification

The Bühlmann and Straub's Credibility model generally estimates premiums, frequencies and severities for policyholders of each non-life insurance company for the next immediate period or year ($n+1$) using annual claim costs measured in Ghana cedi(GHC), annual claim frequencies and severities respectively and their respective exposure units for the years $j=1, 2, 3, \ldots\ldots, n$ for each class risk or company. Where in this paper, j = 1 stands for 2013, and j = 2 stands for 2014, 3 for 2015, 4 for 2016, 5 for 2017 and n stands for the last accounting period of the data (2018) whilst ($n+1$) stands for 2019.

A risk's exposure to loss may vary from period to period and the number of years of observations may also differ over various risks. Let I = 15 be the total number of non-life marine insurance companies represented as risk classes and S_{ij} be the aggregate claim amount in the year j for a class risk or non-life marine insurance company i such that for each of the observed aggregate claims of the company i at period j, there is a corresponding weight W_{ij} representing the policy count; exposure units in the year j for risk i and N_{ij} representing a number of claims in the year j for risk or company i. The Bühlmann and Straub's Credibility theory makes use of a risk parameter Θ which is a random variable and does not observe any specific statistical distribution. In this estimation process, the risk classes stand for the individual companies ($i = 1, 2, 3, \ldots, I$) which experiences losses or pay claims over n-years. Given the fixed risk classes (companies), we use the notation $Y_{ij} = S_{ij} / W_{ij}$ for claim cost and $X_{ij} = S_{ij} / N_{ij}$ for claim severities and $F_{ij} = N_{ij} / W_{ij}$ for claim frequencies for the class risk i in the year j.

Conditional independence according to Bühlmann & Gisler (2005), is an assumption which is frequently appropriate in many situations in insurance practice. We can now estimate $E[\mathrm{F}_{n+1} / \Theta_i] = \lambda_{n+1}(\Theta_i)$ as credibility claim frequency and $E[\mathrm{X}_{n+1} / \Theta_i] = \mu_{n+1}(\Theta_i)$ as credibility claim severity for the year $n+1$ separately for each company respectively. The multiplication of these two estimators will result into a credibility estimate known as credibility frequency-severity claim cost or premium ($P_{n+1}(\Theta_i)$) to be charged policyholders for the year $n+1$ by each company. This is developed on the basis that premiums can be the product of estimated claim frequencies and severities for a particular accounting period.

Assumptions Under Bühlmann-Straub's Credibility Model

The following assumptions according to Pitts, Roger & Susan (2012) and Merwe (2006) form the basis of the Bühlmann-Straub's Credibility Theory. Thus,

i. Given the risk profile Θ_i associated with company i, the Y_{ij} over the n accounting years are independent but not identically distributed either conditionally or unconditionally but the risk profiles are independently and identically distributed.

ii. The class means (hypothetical means); $m(\Theta_i) = E[\mathrm{Y}_{ij} / \Theta_i]$ and the process variance; $s^2(\Theta_i) = W_{ij} Var[\mathrm{Y}_{ij} / \Theta_i]$ are independent of j or the accounting periods whilst expectation of the class means also known as the collective risk mean; $E[m(\Theta)] = E[m(\Theta_i)]$ and the expected process variance; $E[s^2(\Theta)] = E[s^2(\Theta_i)]$ including the variance of the process variance; $\mathrm{var}[m(\Theta)] = \mathrm{var}[m(\Theta_i)]$ are independent of i or the class risks (the individual non-life marine insurance companies).

iii. For different classes of risks or companies ($i \neq d$) the pairs of claim costs and their respective underlined risk profiles ($\Theta_{i,} Y_{il}$) and ($\Theta_d, \mathrm{Y}_{dk}$) are independent.

Theorem: *let* $Y_{i1}, Y_{i2}, Y_{i3}, \ldots\ldots, Y_{in}$ *be a sequence of random variables, each of whose distribution depends on a parameter* Θ_i, *and which, given* Θ_i , *are independent, with* $E[Y_{ij}/\Theta_i] = m(\Theta_i)$ *and* $W_{ij}Var[Y_{ij}/\Theta_i] = s^2(\Theta_i)$, $i = 1,\ldots\ldots\ldots\ldots,n.$

Then the estimator $m_{n+1}(\Theta_i)$ *for which* $E\left[\left(m(\Theta_i) - m_{n+1}(\Theta_i)\right)^2\right]$ *is minimized is given by,*

$$m_{n+1}(\Theta_i) = Z_i\bar{Y}_i + (1 - Z_i)E[m(\Theta)] \tag{1}$$

where

$\bar{Y}_i =$ The average claim cost or hypothetical or class mean for company i

$Z_i =$ Credibility factor for claim histories for company i

$E[m(\Theta)] =$ Collective Risk mean

NB: the model assumptions, theorem and lemma above applied to X_{ij}, s and F_{ij}, s for estimating $X_{n+1}(\Theta_i)$ and $F_{n+1}(\Theta_i)$ respectively.

The proof of the theorem continues at the appendix

$$Z_i = \frac{W_{i\bullet}}{W_{i\bullet} + \dfrac{E[s^2(\Theta)]}{\mathrm{var}[m(\Theta)]}},\ Z_i = [0,1] \tag{2}$$

$$\bar{Y} = E[m(\Theta)] = \frac{\sum_{i=1}^{I} W_{i\bullet}\bar{Y}_i}{W_{\bullet\bullet}} \tag{3}$$

where,

$$W_{i\bullet} = \sum_{j=1}^{n} W_{ij}$$

$$W_{\bullet\bullet} = \sum_{i=1}^{I}\sum_{j=1}^{n} W_{ij}$$

The parameters are therefore estimated as

$$E[s^2(\Theta)] = \frac{1}{I(n-1)}\sum_{i}^{I}\sum_{j}^{n} W_{ij}(Y_{ij} - \bar{Y}_i)^2 \tag{4}$$

$$\mathrm{var}[m(\Theta)] = \frac{1}{W^*}\left\{\frac{1}{In-1}\sum_{i=1}^{I}\sum_{j=1}^{n} W_{ij}(Y_{ij} - \bar{Y})^2 - \frac{1}{I}\sum_{i=1}^{I}\frac{1}{n-1}\sum_{j=1}^{n} W_{ij}(Y_{ij} - \bar{Y}_i)^2\right\} \tag{5}$$

where,

$$W^* = \frac{1}{In-1}\sum_{i=1}^{I} W_{i\bullet}(1-\frac{W_{i\bullet}}{W_{\bullet\bullet}})$$

Model Assumptions under Claim Frequency

Similar to the Bühlmann-Straub model but additional assumptions, we have;

(i) Given Θ_i the N_{ij} (j = 1, 2….,n) are independent and Poisson distributed with Poisson parameter $\lambda_{ij}(\Theta_i) = W_{ij}\Theta_i\lambda$

(ii) The pairs (Θ_1, N_1), (Θ_2, N_2) . . ., are independent, and Θ_1, Θ_2 ….., are independent and identically distributed with E $[\Theta_i] = 1$.

Θ reflects the frequency structure between the risks, whereas λ is the overall frequency level for all class risks or companies.

Given the model assumptions (3.8.1) we have;

$$E[\mathrm{F}_{ij} / \Theta_i] = \Theta_i\lambda \quad (6)$$

$$w_{ij}\ \mathrm{var}[\mathrm{F}_{ij} / \Theta_i] = \Theta_i\lambda = \mathrm{s}^2(\Theta_i) \quad (7)$$

Just like the credibility claim cost, the credibility claim frequency shown by equation (8) is estimated using the same theorem, lemma and assumptions.

$$\lambda_{n+1}(\Theta_i) = Z_i\bar{F}_i + (1-Z_i)\lambda \quad (8)$$

where

$\bar{F}_i$ = The average claim frequency for company i

Z_i = Credibility factor of claim frequencies for company i

λ = The collective mean for claim frequencies

$$\bar{F}_i = \lambda(\Theta_i) = \frac{\sum_{j=1}^{n} W_{ij} F_{ij}}{W_{i\bullet}} \quad (9)$$

$$\lambda = \frac{\sum_{i=1}^{I} w_{i\bullet}\bar{F}_i}{w_{\bullet\bullet}} \quad (9)$$

$$Z_i = \frac{w_{i\bullet}}{w_{i\bullet}+k}, \quad Z_i = [0,1] \quad (11)$$

where,

$$k = \frac{E[\Theta\lambda]}{\mathrm{var}[\Theta\lambda]} = \frac{\lambda}{\lambda^2\ \mathrm{var}[\Theta]} = \frac{1}{\lambda\ \mathrm{var}[\Theta]}$$

$$\mathrm{var}[\lambda(\Theta)] = \lambda^2 \,\mathrm{var}[\Theta] = \lambda^2 (\frac{1}{I}\sum_{i=1}^{I}(\bar{\mathrm{F}}_i - \bar{\mathrm{F}})^2) \qquad (12)$$

where,

$$\bar{F} = \frac{1}{I}\sum_{i=1}^{I}\bar{F}_i$$

Credibility Frequency-Severity Claim Cost

The credibility frequency-severity claims costs; $P_{n+1}(\Theta_i)$ and $\tilde{P}_{n+1}(\Theta_i)$ are given by equation (13) and (14) respectively;

$$P_{n+1}(\Theta_i) = \lambda_{n+1}(\Theta_i).\mu_{n+1}(\Theta_i) \qquad (13)$$

$$\tilde{P}_{n+1}(\Theta_i) = \tilde{\lambda}_{n+1}(\Theta_i).\mu_{n+1}(\Theta_i) \qquad (14)$$

Where $\lambda_{n+1}(\Theta_i)$ is the credibility claim frequency for company *i* in situation where the claim counts or frequencies are said to have follow a Poisson distribution, $\tilde{\lambda}_{n+1}(\Theta_i)$ is also an estimated credibility claim frequency for company *i* with no known distribution for claim counts and $\mu_{n+1}(\Theta_i)$ is credibility claim severity for risk or company *i*.

RESULTS AND DISCUSSIONS

Estimation of credibility claim costs

Table 1 presents the total policy counts ($W_{i\bullet}$), credibility weights (Z_i), average claim cost; $\mathrm{m}(\Theta_i)$ and the credibility claim costs or premiums; $m_{n+1}(\Theta_i)$ (in Ghana cedi (GHC) per policy count for each class risk or company). The credibility weights given by equation (2) for the claim histories of most of the class risks are less than 50% given more weight to the collective class risk mean. Only enterprise insurance company limited and SIC insurance company limited have above 50% weights accredited to their annual claim costs recorded over the six (6) year period whilst that of the remaining risk classes or companies fall below average credibility weight.

The estimated structural parameters that aided in determining the credibility estimates under claim cost; the collective class mean or risk premium; $E[\mathrm{m}(\Theta)]$ the expected process variance; $E[s^2(\Theta)]$ and the variance of the hypothetical means;

$\text{var}[m(\Theta)]$ estimated according to equation (3), (4) and (5) respectively are presented as GHC1 128.96, GHC2 586 182 605.24 and GHC 811 737.34 per policy count respectively.

Table 1: Policy Count, average claim cost, Credibility Factors and Estimates for Claim Cost for Non-life Marine Insurance Companies (Risk Classes)

Marine Insurance company	$W_{i\bullet}$	$m(\Theta_i)$	Z_i	$m_{n+1}(\Theta_i)$
Activa Int. Insurance	650	3715.54	0.169	1567.248
Allianz Insurance	381	5498.77	0.107	1595.708
Donewell Insurance	155	2654.07	0.046	1199.711
Enterprise Insurance	9886	169.865	0.756	403.6203
Ghana Union Assurance	998	1815.8	0.239	1292.788
Glico General Insurance	815	1384.84	0.204	1181.078
Hollard Insurance	758	1741.72	0.192	1246.722
NSIA Ghana Insurance	172	2046.16	0.051	1175.936
Phoenix Insurance	641	207.198	0.168	974.5657
Provident Insurance	236	505.932	0.069	1085.988
Quality Insurance	139	1701.42	0.042	1152.887
RegencyNem Insurance	328	10223.4	0.093	1977.841
SIC Insurance	7785	1399.04	0.710	1320.609
Star Assurance	1064	1377.63	0.250	1191.212
Unique Insurance	479	483.562	0.131	1044.605

Fig. 1 displays bar chart plots of the average claim costs for each non-life marine insurance company. There is a wide variation among the average claim costs of these companies and these variations cannot be attributed to randomness but due to the differences in the risk profiles of the insurance companies. Due to the variation in the risk classes or companies, it is not advisable to neither charge the same minimum premium nor allow policyholders to pay the same premium (collective risk premium) across risk classes since policyholders of other risk classes may have a high tendency of making claim. Moreover, due to small values of the credibility weights accompanying the claim histories of policyholders of the insurance companies or risk classes, less credit is given to the claim data hence, it will not be advisable to do experience rating ($m(\Theta_i)$). The best estimates to be considered in this case is the credibility estimates ($m_{n+1}(\Theta_i)$) which is a balance between the average claim cost (experience rating) and the collective risk mean (class rating) as shown by **Fig. 2**.

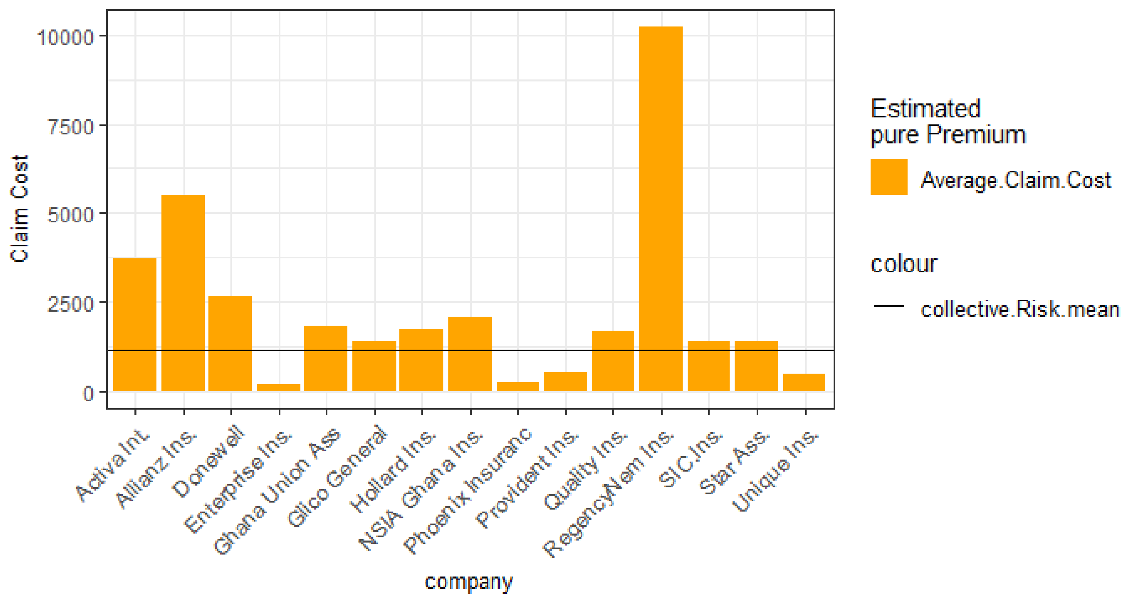

Fig. 1: Bar Chart Plot of the Average Claim costs ($m(\Theta_i)$).

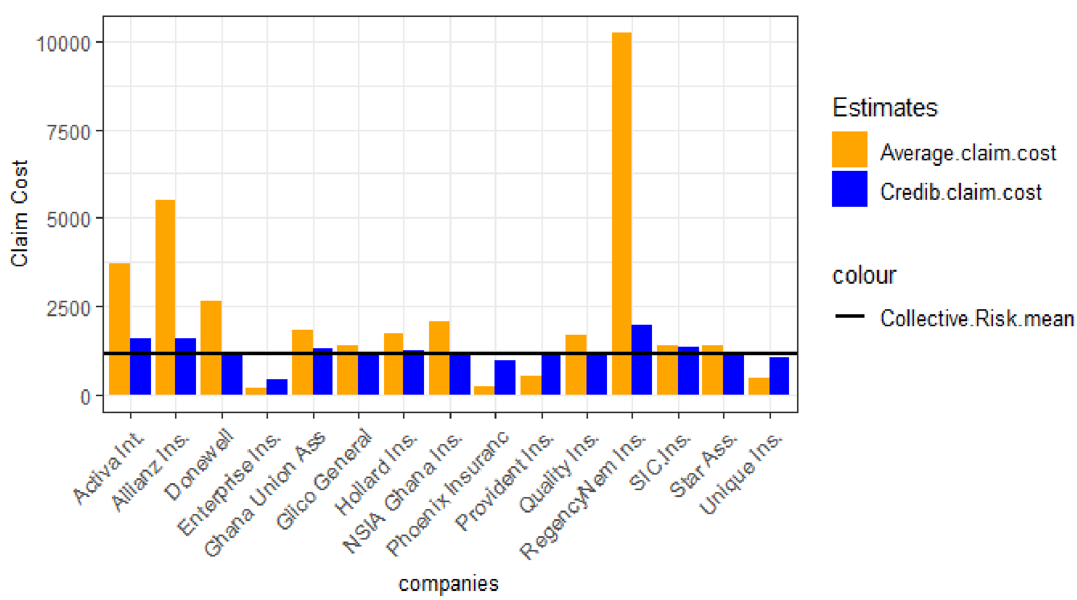

Fig. 2: Bar Chart Plot of Average Claim costs and Credibility claim costs.

Even though the credibility claim cost stands as a better estimate for setting rates we proposed that credibility estimate in terms of the claim frequencies and severities will be ideal for insurance companies for setting better rates because the risk profiles responsible for claim sizes may differ in terms of claim frequency and claim severity. The credibility estimates for claim severities and frequencies are determined and presented below.

Estimation of credibility claim Severities

Table 2 presents claim counts, average claim severities, weights and credibility claim severities for the year, 2019 for the class risks or non-life marine insurance companies. The estimated structural parameters that aided in determining the credibility estimates, the collective class mean or risk premium; $E[\mu(\Theta)]$ the expected process variance; $E[\tilde{s}^2(\Theta)]$ and the variance of the hypothetical means; $\text{var}[\mu(\Theta)]$ are also estimated using equation (3), (4) and (5) respectively and are presented as GHC21232.51, GHC45296617020 and GHC947416672 per claim count respectively.

In **Table 2**, Credibility weights (Z_i) closer to one (1) have their estimated balanced credibility severities ($\mu_{n+1}(\Theta_i)$) closer to their average severities ($\mu(\Theta_i)$) or experience ratings and vice versa. Apart from Activa International Insurance, Enterprise Insurance, Ghana Union Assurance, Regency Nem Insurance and SIC Insurance company limited that have their credibility weights (Z_i) above 50% due to significant values of their claim counts ($N_{i\bullet}$) the rest have low credibility weights.

Table 2: Claim Count, average claim severity, Credibility factors and estimates of claim Severities for the Non-life Marine Insurance Companies

Company	$N_{i\bullet}$	$\mu(\Theta_i)$	Z_i	$\mu_{n+1}(\Theta_i)$
Activa Int. Insurance	54	44724.15	0.53040	41356.225
Allianz Insurance	8	261878.9	0.14334	56502.788
Donewell Insurance	9	45709.00	0.15842	27488.721
Enterprise Insurance	828	2028.121	0.94541	7092.3230
Ghana Union Assurance	49	36983.08	0.50614	34886.660
Glico General Insurance	20	56432.05	0.29494	33551.454
Hollard Insurance	33	40006.67	0.40836	33489.003
NSIA Ghana Insurance	16	21996.25	0.25074	25660.332
Phoenix Insurance	14	9486.714	0.22650	19824.896
Provident Insurance	10	11940.00	0.17298	21833.553
Quality Insurance	13	18192.15	0.21378	24233.977
RegencyNem Insurance	48	69859.77	0.50099	54249.550
SIC Insurance	170	64067.92	0.78049	56170.591
Star Assurance	19	77147.37	0.28439	37856.480
Unique Insurance	11	21056.91	0.18704	22396.001

Estimation of credibility claim frequencies

Table 3 also presents claim count, average claim frequency, weights and estimated credibility claim frequencies for the year, 2019 for the risks or companies. The expected process variance $E[\lambda(\Theta)]$ and the variance of the hypothetical means $\text{var}[\lambda(\Theta)]$ of the claim frequencies are given as 0.05317 and 0.00000375 given by equation (10) and (12) respectively. These structural parameters are obtained with the assumption that the claim counts or frequencies follow a Poisson distribution and since the claim frequency follows a Poisson process, the collective claim frequency (λ) and the expected process variance $E[\lambda(\Theta)]$ have recorded the same value through which $\lambda_{n+1}(\Theta_i)$ and Z_i recorded in **Table 3** were obtained given by equation (8) and (11) respectively. We also estimated the structural parameters $\tilde{\lambda}$, $E[\tilde{\lambda}(\Theta)]$ and the variance of the hypothetical means $\text{var}[\tilde{\lambda}(\Theta)]$ for claim frequencies that do not assume any distribution, given as 0.053171, 0.564098 and 0.00092543 respectively given by the equation (3), (4) and (5) respectively through which we obtained the credibility estimates $\tilde{\lambda}_{n+1}(\Theta_i)$ and $\tilde{Z}_i$ displayed in **Table 3** also given by equation (1) and (2) respectively.

Table 3: Policy Counts, Credibility Claim Frequencies Estimates for the Marine Insurance Companies

Company	$W_{i\bullet}$	$\lambda(\Theta_i)$	Z_i	$\tilde{Z}_i$	$\lambda_{n+1}(\Theta_i)$	$\tilde{\lambda}_{n+1}(\Theta_i)$
Activa Int. Insurance	650	0.0831	0.0439	0.5161	0.0545	0.0686
Allianz Insurance	381	0.021	0.0262	0.3846	0.0523	0.0408
Donewell Insurance	155	0.0581	0.0108	0.2027	0.0532	0.0542
Enterprise Insurance	9886	0.0838	0.411	0.9419	0.0657	0.082
Ghana Union Assurance	998	0.0491	0.0658	0.6208	0.0529	0.0506
Glico General Insurance	815	0.0245	0.0544	0.5721	0.0516	0.0368
Hollard Insurance	758	0.0435	0.0508	0.5543	0.0527	0.0478
NSIA Ghana Insurance	172	0.093	0.012	0.2201	0.0536	0.0619
Phoenix Insurance	641	0.0218	0.0433	0.5126	0.0518	0.0371
Provident Insurance	236	0.0424	0.0164	0.2791	0.053	0.0502
Quality Insurance	139	0.0935	0.0097	0.1857	0.0536	0.0607
RegencyNem Insurance	328	0.1463	0.0226	0.3498	0.0553	0.0858
SIC Insurance	7785	0.0218	0.3546	0.9274	0.0421	0.0241
Star Assurance	1064	0.0179	0.0699	0.6358	0.0507	0.0307
Unique Insurance	479	0.023	0.0327	0.44	0.0522	0.0399

Estimation of credibility Frequency-Severity claim costs

The credibility frequency-severity claim costs $P_{n+1}(\Theta_i)$ and $\tilde{P}_{n+1}(\Theta_i)$ which are product of the credibility claim frequencies and the credibility claim severity given by equation (13) and (14) respectively are recorded in **Table 4**

Table 4: Credibility frequency-severity risk premium

Company	$P_{n+1}(\Theta_i)$	$\tilde{P}_{n+1}(\Theta_i)$
Activa Int. Insurance	2253.209	2837.210
Allianz Insurance	2956.706	2305.086
Donewell Insurance	1463.061	1488.875
Enterprise Insurance	466.2564	581.4190
Ghana Union Assurance	1845.611	1766.749
Glico General Insurance	1731.712	1234.388
Hollard Insurance	1764.259	1601.792
NSIA Ghana Insurance	1376.654	1589.441
Phoenix Insurance	1027.225	735.7420
Provident Insurance	1157.050	1095.110
Quality Insurance	1298.048	1470.142
RegencyNem Insurance	2998.877	4652.793
SIC Insurance	2362.482	1354.393
Star Assurance	1919.483	1162.929
Unique Insurance	1168.695	893.1340

Fig. 3 shows a bar chart plot of the average claim cost, credibility claim cost, the collective risk mean or premium and the two credibility frequency-severity claim costs where the $P_{n+1}(\Theta_i)$ is estimated and represented in the diagram as "cred.F.S" and $\tilde{P}_{n+1}(\Theta_i)$ is shown as "cred.F.S1" whilst **Fig. 4** display a bar chart plot comparing the two credibility frequency-severity claim costs to the credibility claim cost ("Cred.claim.cost"). Fom **Fig. 4** it is obvious that the two credibility frequency-severity claim costs ("Cred.FS" and "cred.FS1") provides estimates higher than the credibility claim costs ("Cred.claim.cost") for majority of the risk classes or companies, an indication that the credibility claim cost underestimated premiums to be charged policyholders for the 2019 accounting period.

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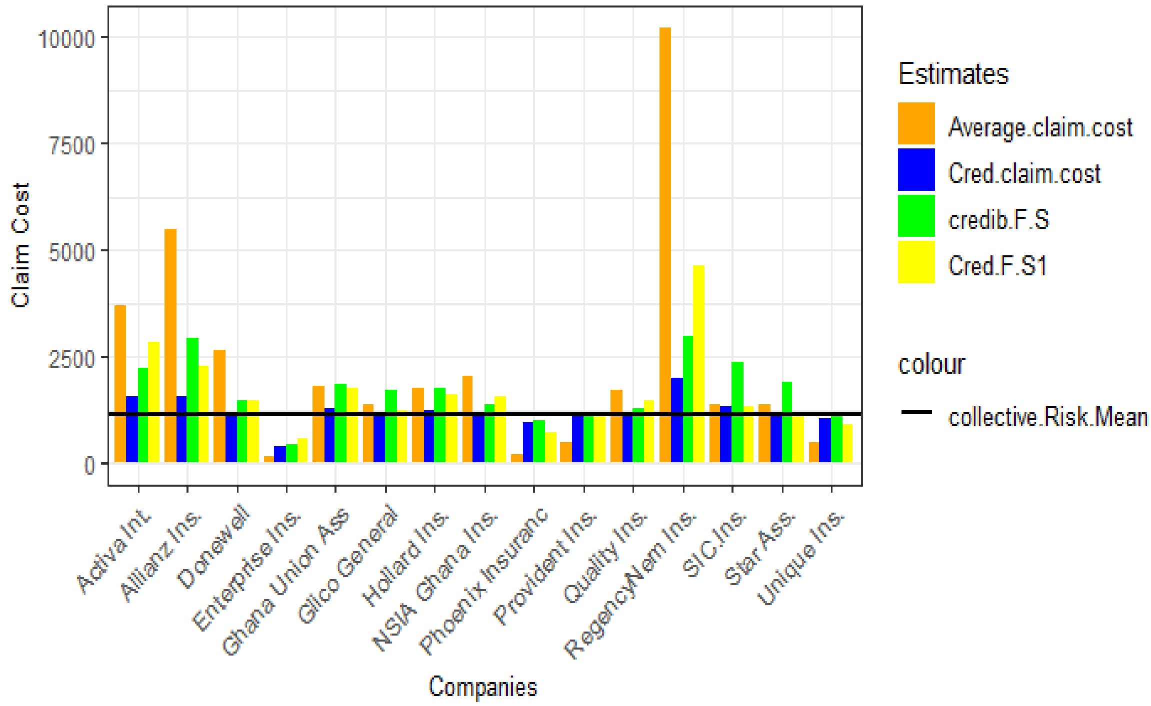

Fig. 3: Bar Chart plot of Average claim cost, Credibility Claim Cost and Credibility Frequency-Severity Claim Costs

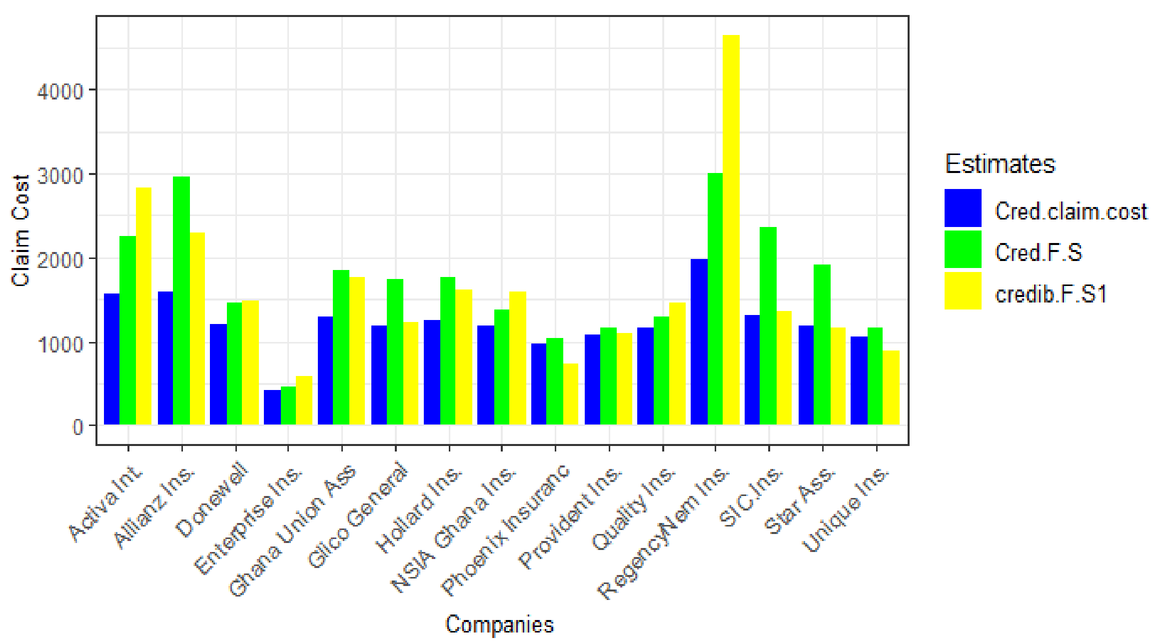

Fig. 4: Bar Chart plot of Credibility Claim Cost and Credibility Frequency-Severity costs.

DISCUSSIONS

The Bühlmann-Straub's Credibility model was used in estimating claim cost or premiums ($m_{n+1}(\Theta_i)$ or "cred.claim.cost") to charge policyholders by the risk classes according to their respective risk profiles for the year 2019. Credibility claim severities and credibility claim frequencies were also determined so as to estimate the severity and frequency of losses for each class risk based on their risk profiles, and the product of the two credibility estimates resulted to credibility frequency-severity claim costs or premiums; $P_{n+1}(\Theta_i)$ and $\tilde{P}_{n+1}(\Theta_i)$ for claim counts assuming a Poisson distribution and no distribution respectively for the year 2019 recorded in **Table 4**.

In line with the argument of this paper, based on the risk profiles underlying the claim frequency and claim severity we established that the credibility frequency-severity claim costs provide a better balance between experience rating and class rating compared to the credibility claim cost. As argued by Kagen (2018) and Insurance Institute of Michgan (2019) that factors responsible for claim frequency and claim severity for insurance companies are different though the resultant effect of the two factors is the claim size or amount. The credibility claim frequency and severity which are predicted claim frequencies and predicted claim severities for policyholders of class risks for the year 2019, suggested a premium higher than the premium predicted by the credibility claim cost as presented by **Fig. 4**. Thus, $P_{n+1}(\Theta_i)$ and $\tilde{P}_{n+1}(\Theta_i)$ are estimates above $m_{n+1}(\Theta_i)$ for all risk classes except for phoenix insurance company and Unique insurance company because $\tilde{\lambda}_{n+1}(\Theta_i)$ decreases for these two non-life marine insurance companies compared to their respective $\lambda_{n+1}(\Theta_i)$ as shown in **Table 3**.

Fellingham, Tolley & Herzog (2005) comparison of credibility estimates of health insurance claim costs and their conclusion about the under estimation of claim costs by estimates of Bayesian credibility models is in sharp contrast to the outcome of this study after determining the credibility estimates of claim frequency and severity separately and independently and their product yielding the credibility frequency-severity estimate for claim costs.

This shows that the minimum premium determination in using inadequate claim histories among individual class risks using credibility models must incorporate the frequency and severity of losses so as to be able to adequately cover losses transferred by policyholders to insurers. It is therefore, our contention that claim frequencies and severities be considered in determining credibility estimates as minimum rates for pricing non-life insurance products.

CONCLUSIONS AND RECOMMENDATIONS

In accordance with the objectives of this study, the researchers have shown which credibility estimates underestimate claim cost and justify why the credibility frequency-severity claim costs should be considered over the credibility claim cost as a balance between the experience rating and class rating of risks.

The credibility frequency-severity claim costs; both $P_{n+1}(\Theta_i)$ and $\tilde{P}_{n+1}(\Theta_i)$ are estimates for claim costs or the average claim cost for the individual class risks just like the credibility claim cost; $m_{n+1}(\Theta_i)$ but for all risk classes except for phoenix insurance company and Unique insurance company in the case of $\tilde{P}_{n+1}(\Theta_i)$ (instance where claim counts does not follow any distribution), the credibility claim estimates; $m_{n+1}(\Theta_i)$ underestimate the claim cost (the average claim cost) compared to $P_{n+1}(\Theta_i)$ and $\tilde{P}_{n+1}(\Theta_i)$ because $m_{n+1}(\Theta_i)$ did not consider the influence of the claim frequency and severity that are also affected by different risk profiles. Therefore, the estimation of $m_{n+1}(\Theta_i)$ with a common underlying risk profile Θ though different across class risks is affected by increase in the expected process variance and decrease the credibility weights of the experience rating and in effect undermines the claim costs or premiums of the insurance companies.

To avoid denial of fund for investment and the collapse of the insurance companies due to underestimation of risk premiums in situations where there are limited claim histories available on a risk, we recommend that claim frequencies and severities and their respective risk profiles be considered and always included in the credibility rate makings by insurance companies.

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Appendix

The **Theorem** stated above which is a direct consequence of **Theorem 4.14** in (Pitts, Roger & Susan, 2012) implies that to find the credibility estimator, $m_{n+1}(\Theta_i)$ which is a linear function of the annual claim cost of a risk ($Y_{i1}, Y_{i2}, \ldots, Y_{in}$) with minimum mean square error from the pure premium; there is the need to estimate $\alpha_{i1}, \alpha_{i2}, \ldots, \alpha_{in}$ that will optimize the credibility estimator ($m_{n+1}(\Theta_i)$) of $m_n(\Theta_i)$ given by;

$$m_{n+1}(\Theta_i) = \alpha_{i0} + \alpha_{i1}Y_{i1} + \alpha_{i2}Y_{i2} + \cdots + \alpha_{in}Y_{in} = \alpha_{i0} + \sum_{j}^{n} \alpha_{ij}Y_{ij} \qquad (1)$$

Applying the **Lemma** below which are direct consequence of **Theorem 4.13** in (Pitts, Roger & Susan, 2012) we estimate $\alpha_{i1}, \alpha_{i2}, \ldots, \alpha_{in}$ in equation (2) and subsequently obtain the optimized credibility estimator ($m_{n+1}(\Theta_i)$) of $m(\Theta_i)$.

Lemma

(i) $E[Y_{ij}] = E[\bar{Y}_i] = E[m(\Theta)]$

(ii) $E[Y_{ij}\, m(\Theta_i)] = E[m^2(\Theta_i)]$

(iii) $E[Y_{ij}\, Y_{ik}] = E[m^2(\Theta)]$ *For* $j \neq k$;

(iv) $E[Y_{ij}^2] = \frac{1}{W_{ij}} E[s^2(\Theta)] + E[m^2(\Theta)].$

These constants $(\alpha_{i1}, \alpha_{i2}, \ldots, \alpha_{in})$ that minimizes Q in equation (2) under Bühlmanns-Straub's credibility theory are not equal because Y_{ij}, s are not identically distributed.

$$Q = E\left[\left(m(\Theta_i) - \mathrm{m}_{n+1}(\Theta_i)\right)^2\right] = E\left[\left(m(\Theta_i) - \alpha_{i0} - \sum_{j=1}^{n} \alpha_{ij}Y_{ij}\right)^2\right] \qquad (2)$$

Taking a partial derivative of Q with respect to α_{i0} we have,

$$\partial Q / \partial \alpha_{i0} = 0 = E\left[\left(m(\Theta_i) - \alpha_{i0} - \sum_{j=1}^{n} \alpha_{ij}Y_{ij}\right)\right]$$

$$\alpha_{i0} = E[m(\Theta_i)] - \sum_{j=1}^{n} \alpha_{ij} E[Y_{ij}]$$

$$\alpha_{i0} = (1 - \sum_{j=1}^{n} \alpha_{ij}) E[m(\Theta_i)] \qquad (3)$$

From equation (2) we obtained equation (4)

$$Q = E\left[Y_{ik}\left(m(\Theta_i) - \alpha_{i0} - \alpha_{ik}Y_{ik} - \sum_{j\neq k}^{n}\alpha_{ij}Y_{ij}\right)\right] \tag{4}$$

Estimating the value of α_{ik} from equation (4) by taking partial derivative of Q with respect to α_{ik}

$$\partial Q \big/ \partial\alpha_{ik} = 0 = E\left[Y_{ik}\left(m(\Theta_i) - \alpha_{i0} - \alpha_{ik}Y_{ik} - \sum_{j\neq k}^{n}\alpha_{ij}Y_{ij}\right)\right]$$

$$E[m(\Theta_i)\,\mathrm{Y}_{ik}] - E[\alpha_{i0}Y_{ik}[-E[\alpha_{ik}Y_{ik}{}^2] - \sum_{\substack{j=1\\ j\neq k}}^{n} E[\alpha_{ij}Y_{ij}Y_{ik}] = 0$$

$$E[m^2(\Theta)] - \alpha_{i0}E[m(\Theta)] - \frac{\alpha_{ik}}{W_{ik}}E[s^2(\Theta)] - \alpha_{ik}[m^2(\Theta)] - \alpha_{ik}\sum_{j\neq k}^{n}E[m^2(\Theta)] = 0$$

$$NB: \quad \alpha_{ik} + \sum_{\substack{j=1\\ j\neq k}}^{n}\alpha_{ij} = \sum_{j=1}^{n}\alpha_{ij}$$

$$E[m^2(\Theta)] = \alpha_{i0}E[m(\Theta)] + \sum_{j=1}^{n}\alpha_{ij}E[m^2(\Theta)] + \frac{\alpha_{ik}}{W_{ik}}E[s^2(\Theta)]$$

Substituting the estimate of α_{i0} from equation (3)

$$E[m^2(\Theta)] = (1 - \sum_{j=1}^{n}\alpha_{ij})E[m(\Theta)]^2 + \sum_{j=1}^{n}\alpha_{ij}E[m^2(\Theta)] + \frac{\alpha_{ik}}{W_{ik}}E[s^2(\Theta)]$$

$$E[m^2(\Theta)] - E[m(\Theta)]^2 + \sum_{j=1}^{n}\alpha_{ij}E[m^2(\Theta)] - \sum_{j=1}^{n}\alpha_{ij}E[m(\Theta)]^2 = \frac{\alpha_{ik}}{W_{ik}}E[s^2(\Theta)]$$

$$(1 - \sum_{j=1}^{n}\alpha_{ij})\,\mathrm{var}[m(\Theta)] = \frac{\alpha_{ik}}{W_{ik}}E[s^2(\Theta)]$$

$$\alpha_{ik} = (1 - \sum_{j=1}^{n}\alpha_{ij})\frac{\mathrm{var}[m(\Theta)]W_{ik}}{E[s^2(\Theta)]}$$

$$\sum_{k=1}^{n}\alpha_{ik} = \sum_{j=1}^{n}\alpha_{ij}$$

$$\sum_{j=1}^{n}\alpha_{ij} = \frac{\operatorname{var}[m(\Theta)]\sum_{j=1}^{n}W_{ij}}{E[s^2(\Theta)] + \operatorname{var}[m(\Theta)]\sum_{j=1}^{n}W_{ij}}$$

$$\sum_{j=1}^{n}\alpha_{ij} = \frac{\sum_{j=1}^{n}W_{ij}}{\sum_{j=1}^{n}W_{ij} + \frac{E[s^2(\Theta)]}{\operatorname{var}[m(\Theta)]}} \tag{5}$$

Putting equations (3) and (5) into equation (1) to find the optimized credibility estimator in equation (6) that minimized Q (the mean square error).

$$m_{n+1}(\Theta_i) = Z_i\bar{Y}_i + (1 - Z_i)E[m(\Theta)] \tag{6}$$